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SOME PROBLEMS OF THE PHYSICAL THEORY OF METEORS

V. N. Lebedinets and Yu. I. Portnyagin[1]

ABSTRACT: The effective cross-sections of diffusion of meteor atoms and molecules are determined and applied to problems of the physical theory of meteors. One possible mechanism of fragmentation of small meteor bodies leading to the appearance of radiometeors is considered.

Thus far, investigators who are concerned with the physical theory of /9* meteors do not share a common opinion on the size of the effective diffusion cross-section of meteor atoms in the atmosphere. Q_d and its dependence on the velocity of evaporated meteor atoms. Various investigators [1-5] use values of Q_d in their papers which differ by more than one order of magnitude. As far as we know, there are no experiments on the measurement of the diffusion cross--section of typical meteor atoms in the atmosphere at meteor velocities. The present paper gives the results of calculation of Q_d for collisions of typical meteor atoms with atmospheric molecules.

At meteor velocities, the formula for Q_d has the form:

$$Q_d = 2\pi \int_0^\infty (1-\cos\theta) p\,dp, \qquad (1)$$

where p is the sighting parameter,

$$\theta = \pi - 2 \int_{R_0}^{\infty} \frac{p\,dR}{R^2 \sqrt{1-U(R)/E_0-(P/R)^2}} . \qquad (2)$$

Here, E_0 is the initial relative kinetic energy of the colliding particles, U(R) is the interaction potential, R is the internuclear distance, and R_0 is the distance of maximum approach.

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*Numbers in the margin indicate pagination in the foreign text.

We will assume that: 1) the atoms are described by the statistical model of Thomas-Fermi-Dirac; 2) the electron shells of the interacting atoms are rather considerably overlapped; 3) the electron density of the quasimolecules which are formed is a simple superposition of the electron densities of non- /10
-interacting atoms; 4) the outer electrons cause an insignificant distortion of the sphericity of the electron envelopes. Then, using the minimum and maximum principles, we can obtain the following expression for the minimum and maximum energy of a system of two interacting atoms [6-8]:

$$H_{min}=\varkappa_k\int[(\rho_{01}+\rho_{02})^{2/3}-\frac{5}{6}(\rho_{01}^{2/3}+\rho_{02}^{2/3})](\rho_{01}+\rho_{02})d\tau-$$
$$-\frac{1}{2}e^2\int\left(\frac{Z_1}{\vec{r}_1}+\frac{Z_2}{\vec{r}_2}\right)(\rho_{01}+\rho_{02})d\tau-\frac{1}{2}V_0e\int(\rho_{01}+\rho_{02})d\tau- \quad (3)$$
$$-\varkappa_a\int[(\rho_{01}+\rho_{02})^{1/3}-\frac{2}{3}(\rho_{01}^{1/3}+\rho_{02}^{1/3})](\rho_{01}+\rho_{02})d\tau,$$

$$H_{max}=\varkappa_k\int\left[\frac{5}{6}(\rho_{01}+\rho_{02})-\frac{2}{3}(\rho_{01}^{2/3}+\rho_{01}^{2/3})^{3/2}\right](\rho_{01}^{2/3}+\rho_{02}^{2/3})d\tau-$$
$$-\frac{1}{2}e^2\int\left(\frac{Z_1}{\vec{r}_1}+\frac{Z_2}{\vec{r}_2}\right)(\rho_{01}+\rho_{02})d\tau-\frac{1}{2}V_0e\int(\rho_{01}+\rho_{02})d\tau- \quad (4)$$
$$-\varkappa_a\int\left[\frac{2}{3}(\rho_{01}+\rho_{02})-\frac{4}{3}\left(\rho_{01}^{2/3}+\rho_{02}^{2/3}\right)^{3/2}\right](\rho_{01}^{1/3}+\rho_{02}^{1/3})d\tau,$$

where- $\varkappa_k=\frac{3}{10}\left(3\pi^2\right)^{2/3}e^2a_0\approx 2.9\ e^2a_0$; $\varkappa_a=\frac{4}{3}\left(3/\pi\right)^{1/3}e^2\approx 0.73\ e^2$;

V_0 is the Lagrangian multiplier; ρ_{01}, ρ_{02} are the electron densities of the non-excited first and second atoms, respectively; $\vec{r}_1$, $\vec{r}_2$ are the radius vectors directed from the first and second nuclei, respectively.

Determining the interaction potential as the sum of the value of the change in the electron energy of the system of two atoms with approach of the atoms to a distance R and the value of the Coulomb interaction at this distance, and considering that the true potential is equal to the average of the minimum and maximum potentials, we can obtain from (3) and (4)

$$U(R)=\frac{Z_1Z_2e^2}{2R}\left[\psi\left(Z_1^{1/3}R/a\right)+\psi\left(Z_2^{1/3}R/a\right)\right]+$$
$$+\frac{\pi}{3R}\int_{\max(0,R-r_{b1})}^{\min(r_{b2},R+r_{b2})}dr_2\int_{|R-r_2|}^{\min(r_{b1},R+r_2)}r_1r_2F(r_1,r_2)\,dr_1 \quad (5)$$

where $\psi(Z\ 1/3\ R/a)$ is a Thomas-Fermi-Dirac screening function: r_{b1}, r_{b2} are the limiting radii of the atoms; Z_1, Z_2 are the charges of the first and second nuclei:

$$F(r_1 r_2)=\{\varkappa_k[(\rho_{01}+\rho_{02})^{5/3}-(\rho_{01}^{5/3}+\rho_{02}^{5/3})]-2\varkappa_d[(\rho_{01}+\rho_{02})^{4/3}-(\rho_{01}^{4/3}+\rho_{02}^{4/3})]\}.$$

By numerical integration of expression (5) on the BESM computer, we obtained the interaction potentials of all typical meteor atoms with N and O atoms. Values ρ and ψ were obtained by interpolation from the four-point Lagrangian formula for the values of ρ and ψ given in [9]. The results of the calculations may be approximated by the expression:

$$U(R) \approx \alpha\ exp\ (-\beta R), \tag{6}$$

where α and β are constants for each pair of atoms.

In order to determine the interaction potential of the atom-molecule system, we suggest that: 1) the molecule may be viewed as a rigid system of dumbbell type; 2) the average interaction potential of the atom-molecule system may be obtained from the orientation-dependent potential by direct averaging over all possible orientations of the molecule; 3) the potentials of the atom--atom system provide independent contributions to the orientation-dependent potential. /11

With these assumptions, and taking (6) into account, it is easy to obtain the analytical expression for the interaction potential of the atom-molecule:

$$U(\rho)=\frac{2\alpha e}{d\beta}^{-\beta\rho}\left[\ \mathrm{sh}(\beta d)\left(1+\frac{1}{\beta\rho}\right)-\mathrm{ch}(\beta d)\frac{d}{\rho}\right]. \tag{7}$$

where d is half of the interatomic distance in the molecule, and ρ is the distance between the nucleus of the atom and the center of inertia of the molecule.

Substituting (7) into (1) and performing the numerical integration, we find the effect of diffusion cross-sections of the meteor atoms in the atmosphere. The values obtained for Q_d are somewhat greater than the results of the calculation performed by H. F. Maffey and D. W. Sida [3, 4]. This is apparently explained by the fact that these authors used a formula for calculating Q_d which was derived from the minimum principle, and thereby obtained only a low estimate of the possible interaction. The difference between our calculations and the values of Q_d which were found by B. Yu. Levin and L. A. Manning is explained by the fact that the interaction of the atoms and molecules at meteor velocities has an essentially different nature in comparison to the interaction under gas-kinetic conditions. The difference between our results and the results obtained in [5] have to do with the lack of a basis for the selection of a formula for Q_d in [5]. The use of a formula for estimating Q_d which was obtained for electron scattering on atoms (first Born approximation) is unjustifiably rough at the present time.

The results of our calculations may be approximated by a formula which is suitable for practical utilization:

$$Q_d \approx \frac{c}{v} = \frac{1.7 \cdot 10^{-9}}{v}, \qquad (8)$$

where v is the relative velocity in cm/sec. The values of Q_d, calculated in [3, 4] may be approximated by a relationship of the same type.

The values of U(R) and Q_d which were obtained make it possible to examine a number of problems in the physical theory of meteors (coefficient of accommodation, coefficient of obstruction, and the initial expansion of the meteor track, etc.).

In the physical theory of meteors, the accommodation coefficient K for stony meteor bodies is usually assumed to be equal to 1 [1]. For iron meteor bodies B. Yu. Levin obtained $K \approx 0.75$. Using the value which we obtained for U(R), we can find the value of K for iron meteor bodies.

At energies of hundreds of electron volts, the collision of molecules with the surface of a solid may be viewed within the framework of a model

of individual collisions [10, 11], introducing the effective scattering parameters [12]:

$$m_{\text{eff}} = m\left(1+\frac{W}{E_0}\right),\ P_{\text{eff}} = P\left(1+\frac{W}{E_0}\right)^{-1/2},\ E_{\text{eff}} = E_0\left(1+\frac{W}{E_0}\right). \tag{9}$$

Here m is the reduced mass, W is the depth of the potential depression, in which the solid is located (~20 to 30 eV).

In determining the probability of collision of flying particles with surfaces or internal atoms of a solid and spherically-symmetrical scattering in an element of the solid angle $d\Omega = 2\pi \sin\theta d\theta$ (θ being the scattering angle in a laboratory system of coordinates), we can obtain the following expression for the coefficient of accommodation on any flat surface: /12

$$k = 1 - \frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{\varkappa\cos\theta - \sqrt{1-\varkappa^2\sin^2\theta}}{(1+\varkappa)^2}\left[\sigma + \frac{(1-\sigma)\cos\theta}{1+\cos\theta}\right]\left(2\varkappa\cos\theta + \frac{(1+\varkappa^2\cos 2\theta)}{\sqrt{1-\varkappa^2\sin^2\theta}}\sin\theta\right)d\theta, \tag{10}$$

where χ is the ratio of the masses of the colliding particles, σ is the section of elastic back-scattering ($\sigma = \pi p_0^2$). To determine the sighting parameter p_0 corresponding to the back-scattering, it is easy to obtain the following expression:

$$\int_{R_0}^{\infty} \frac{p_0 dR}{R^2\sqrt{1-U(R)/E_0-(p/R)^2}} = \frac{\pi - \arccos(-\varkappa)}{2}. \tag{11}$$

Solving equation (11) for p_0 and substituting the values obtained for σ in (10) and performing integration, we can find K.

With an energy of 50 eV, $\sigma = 0.98\pi a_0^2$ (a_0 being the Bohr radius), K = = 0.89. As the energy increases, σ decreases. Moreover, at high energies we can disregard (9). Hence, as E_0 increases the value of K approaches 1. We can therefore consider that the coefficient of accommodation for both stony and iron meteor bodies is practically equal to 1.

Now let us consider the blockage of a circular cylinder with a flat end surface by particles evaporated from the center of the end surface, giving a maximum blocking effect. The evaporated particles undergo their first

collision with the molecules of the atmosphere on the average after a time Δt following evaporation:

$$\Delta t = \frac{\lambda}{v} = \frac{\mu_a m_H}{Q_d \rho v}, \tag{12}$$

where λ is the free path length of the evaporating particles in the atmosphere, μ_a is the molecular weight of the atmospheric molecules, m_H is the mass of a hydrogen atom and ρ is the atmospheric density. After time Δt the particles are separated from the axis of the cylinder by a distance

$$l_t = \frac{\mu_a m_H \sin\gamma}{Q_d \rho} \cdot \frac{v_a}{v}, \tag{13}$$

where v_a is a velocity corresponding to the surface temperature of the body; γ is the angle with the normal to the end surface. From (13) we can find the maximum value of γ at which a particle still is involved in blockage (undergoes its first collision in front of the end surface)

$$\sin\gamma_0 = \frac{Q_d r \rho^2}{\mu_a m_H} \frac{v}{v_a}; \tag{14}$$

Here r is the radius of the cylinder.

Consequently, of the total number N of evaporated particles, the following are involved in blockage:

$$N_0 = \int_0^{2\pi} \int_0^{\gamma_0} dN(\gamma) = 2N \int_0^{\gamma_0} \sin\gamma\cos\gamma d\gamma = N\sin^2\gamma_0. \tag{15}$$

Each of the N_0 particles can protect the end surface from the collision of one of the incident air molecules. The number of molecules of air striking the end surface per second is /13

$$N_a = \frac{\pi r^2 \rho v}{\mu_a m_H}. \tag{16}$$

From (15), (16) and (8) we can find the expression for the blocking coefficient:

$$1 \quad \zeta = \frac{N_0}{N_a} = \frac{c^2 N \rho}{\pi \mu_a m_H v v_a^2}. \tag{17}$$

During intensive evaporation, when practically all of the energy received by the meteor body is expended in its evaporation, formula (17) assumes the form

$$1 \quad \zeta = \frac{\Lambda c^2 \rho^2 v^2 r^2}{2 \mu \mu_a m_H^2 Q v_a^2}, \tag{18}$$

where Q is the energy required to evaporate one gram of meteor substance and μ is the molecular weight of the meteor substance.

We can disregard blockage if $1 - \zeta \leqslant 0.1$. From (18) we find the value of r at which blockage is low:

$$r \leqslant \frac{\sqrt{0.2 \mu \mu_a m^2_H Q v_a^2}}{\Lambda^{1/2} c \rho v}. \tag{19}$$

If we compare the results we have obtained with those obtained by B. Yu. Levin [1], we find that the minimum value for the mass of the meteor bodies at which it is necessary to take into account the effect of blockage, according to our calculations, with an average velocity of meteors being considered as 40 km/sec, exceeds the value for the minimum mass according to B. Yu. Levin by more than four orders of magnitude. Inasmuch as we estimated the maximum blocking effect, such a difference may be explained by the fact that B. Yu. Levin failed to calculate the relationship of Q_d to the velocity, and the value of Q_d was assumed to be equal to a value at gas-kinetic conditions. In addition, he failed to consider that each evaporated particle may theoretically or may actually protect the end surface of a meteor body from collision with only one air molecule.

Hence, we come to the conclusion that for the majority of radar-located meteors, the blocking effect may be disregarded.

The value of the initial radius of the ionized meteor tracks R_n was first estimated in [2]. Here, however, a number of unfounded assumptions were made:

no allowance was made for the fact that there is a dependence of the effect of diffusion cross-section Q_d on the velocity of the colliding particles; the value Q_d was assumed to be the same as for gas-kinetic conditions; the value of Q_d for ions was assumed to be five times greater than for neutral particles; it was assumed that prior to establishment of thermal equilibrium, each of the evaporated particles undergoes a very high number of collisions. In subsequent works [13, 14] calculation of R_n was conducted with consideration of the dependence of Q_d upon the velocity. The value of Q_d was taken from [3]. However, the assumption regarding a very high number of collisions was retained. Until the velocity of the evaporating particles decreases to thermal velocity, the particle undergoes on the average 10 to 12 collisions. Due to the relationship of Q_d to the relative velocity of the colliding particles, the resultant free path length of meteor atoms and ions rapidly decrease. Therefore, the principal contribution to R_n is made by the first few paths. Hence, the assumption that was made in the preceding papers regarding the large number of collisions prior to establishment of thermal equilibrium is not justified. /14

By performing some simple geometric constructions, it is easy to show that after four collisions with atmospheric molecules a meteor atom or ion is located on the average at a distance of $\overline{R}_n$ from the axis of the track such that

$$\overline{R}_{n_\infty}^{-2}=\int_{-\pi}^{\pi}\{l_1^2\sin^2\theta+l_2^2\sin^2\theta[\sin^2\varphi_1+\cos^2\theta(1+\cos\varphi_1)^2]+2l_1l_2\sin^2\theta\cos\theta(1+\cos\varphi_1)\}d\varphi_1+$$

$$+\frac{1}{2\pi^2}\int_{-\pi}^{\pi}d\varphi_1\int_{-\pi}^{\pi}d\varphi_2\frac{l_1l_3\sin^2\theta\cos\theta(1+\cos\varphi_1)[\cos^2\theta(1+\cos\varphi_1)+\cos\varphi_2(\cos^2\theta-\sin^2\theta\cos\varphi_1)]}{\sqrt{(\cos^2\theta+\sin^2\theta\cos^2\varphi_1[\sin^2\varphi_1+\cos^2\theta(1+\cos\varphi_1)^2]}}+$$

$$+\frac{1}{2\pi^2}\int_{-\pi}^{\pi}d\varphi_1\int_{-\pi}^{\pi}d\varphi_2\frac{l_2l_3\sin^2\theta[\cos^2\theta(1+\cos\varphi_1)+\cos\varphi_2(\cos^2\theta-\sin^2\theta\cos\varphi_1)]\cos^2\theta(1+\cos\varphi_1)^2}{\sqrt{(\cos^2\theta+\sin^2\theta\cos^2\varphi_1[\sin^2\varphi_1+\cos^2\theta(1+\cos\varphi_1)^2]}}+ \quad (20)$$

$$+\frac{1}{2\pi^2}\int_{-\pi}^{\pi}d\varphi_1\int_{-\pi}^{\pi}d\varphi_2\frac{l_2l_3\sin^2\theta\sin^2\varphi_1[\cos^2\theta(1+\cos\varphi_1)+\cos\varphi_2(\cos^2\theta-\sin^2\theta\cos\varphi_1)]}{\sqrt{(\cos^2\theta+\sin^2\theta\cos^2\varphi_1[\sin^2\varphi_1+\cos^2\theta(1+\cos\varphi_1)^2]}}+$$

$$+\frac{1}{4\pi^2}\int_{-\pi}^{\pi}d\varphi_1\int_{-\pi}^{\pi}d\varphi_2l^2{}_3\sin^2\theta\left\{\sin^2\varphi_2+\frac{[\cos^2\theta(1+\cos\varphi_1)+\cos\varphi_2(\cos^2\theta-\sin^2\theta\cos\varphi_1)]^2}{\cos^2\theta+\sin^2\theta\cos^2\varphi_1}\right\},$$

where l_1, l_2, l_3 are the free path lengths after the first, second and third collisions; θ is the angle at which the particle is deflected following a collision in the laboratory system of coordinates; φ_1 and φ_2 are the azimuth angles which take into consideration the equiprobability of movement of the atom following a collision along any of the cones which are formed with an aperture angle θ and a height which is oriented according to the velocity of the atom or ion prior to collision.

Performing integration in (20), we obtain the following expressions for $\overline{R}_{n_0}$:

$$\overline{R}_{n_0}^{-2}=\sin^2\theta\{l_1^2+\frac{1}{2}l_2^2(1+3\cos^2\theta)+2l_1l_2\cos\theta+\frac{l_3\cos^2\theta}{\pi}[(l_1\cos\theta+l_2\cos^2\theta+ \\ +l_2)I_1+(2l_1\cos\theta+3l_2\cos^2\theta+l_2)I_2+(l_1\cos\theta+3l_2\cos^2\theta-l_2)I_3+(l_2\cos^2\theta-l_2)I_4]+ \\ +l_3^2\left[\frac{1}{2}+\frac{3}{2}\cos^2\theta+(\operatorname{ctg}^2\theta-\cos^2\theta+\frac{1}{2}\sin^2\theta)(1-\cos\theta)\right]\}, \qquad (21)$$

where

$$I_k=\int_{-\pi}^{\pi}\frac{\cos^{k-1}\varphi_1 d\varphi_1}{\sqrt{a_1\cos^4\varphi_1+a_2\cos^3\varphi_1+a_3\cos^2\varphi_1+a_4\cos\varphi_1+a_5}};$$

The velocity of the evaporated particles following χ^1 collisions in the case of spherically-symmetrical scattering and equality of the masses of the colliding particles will be equal to

$$v_{\chi^1}=v_c\exp(-0.406\chi^1). \qquad (22)$$

Assuming $\overline{\cos\theta}$ = 2/3, with the aid of (8), (21) and (22) we can find $\overline{R}_{n_0}$ = /15
= 0.93 l_0.

The value of $\overline{R}_{n_0}$ which we obtained gives the minimum value of R_n. The maximum value of the initial radius may be estimated by adding to $\overline{R}_{n_0}$ all the other paths prior to establishment of thermal equilibrium with the atmosphere.

Hence, we find that the actual value for the initial radius lies within the limits

$$0.93l_0 < R_n < 1.5l_0. \tag{23}$$

Comparing (23) with the results of radar measurements of the initial radius of the ionized meteor tracks, conducted at Khar'kov [13], Kiev [15] and Jodrell Bank [16], indicates good agreement of theory with experiment.

Now let us examine the deformation and breakup of fine meteor bodies. At the present time, the majority of investigators who deal with meteors feel that the breakup of meteor bodies which is observed is due to their extremely brittle structure. Oepic [17] proposed an alternative mechanism for breakup, according to which the fine meteor bodies are melted thoroughly as they move through the atmosphere, deformed and then breakup; however, due to the contradiction which Oepic found with observations, he was forced to abandon this breakup mechanism in favor of a hypothesis of very brittle meteor bodies.

In this paper, he conducted a very strict examination of the method of deformation and breakup of molten meteor bodies. He showed that the important features of weak photographic meteors with 0 to +4 stellar magnitudes [18, 19, 20] may be explained by deformation and breakup of molten meteor bodies, without reference to the hypothesis of the unusually fragile structure of meteor bodies.

Let us examine the heating of a spherical meteor body as it moves through the atmosphere. The solution of the thermal conductivity equation for such a case was obtained in [21] and has the form (without considering braking):

$$\tau(r,\rho) = \frac{A\rho r_0 v^3}{8\lambda\left[\frac{1}{x_0}\operatorname{ch}\left(\frac{r_0}{x_0}\right) - \frac{1}{r_0}\operatorname{sh}\left(\frac{r_0}{x_0}\right)\right]} \cdot \frac{1}{r}\operatorname{sh}\left(\frac{r}{x_0}\right);\ x_0 = b\sqrt{\frac{H}{v\cos z}}, \tag{24}$$

where $\tau(r, \rho)$ is the temperature calculated from the initial temperature of the body T_0, b is the coefficient of thermal conductivity, A is the coefficient of heat transmission, r_0 is the radius of the body, λ is a coefficient of

thermal conductivity, H is the height of a uniform atmosphere and z is the zenith distance of the radiant.

The latent heat of fusion of the meteor substance is approximately equal to the energy required to heat it from 0°K to $T_0 \approx 280°K$, so that in (24) with $T > T_{fusion}$ (T_{fusion} being the fusion temperature) it is better to calculate the temperature from 0°K.

The temperature of the surface of the body

$$\tau(r_0,\rho)=\frac{\Lambda r_0 v^3}{8\lambda\left[\frac{r_0}{x_0}\operatorname{cth}\left(\frac{r_0}{x_0}\right)-1\right]}. \qquad (25)$$

The rate of evaporation of the substance in a vaccum may be described as follows:

$$\log(\Delta M)=-4.23+c_1+\frac{1}{2}\log\mu-\frac{c_2}{T}-\frac{1}{2}\log T; \qquad (26)$$

Here ΔM is the mass of the substance evaporating from a unit surface in a unit time; c_1 and c_2 are constants. /16

Equating the energy given off to evaporation at τ of the formula, to the energy transmitted to the meteor body by collisions of atmospheric molecules with it, we find the atmospheric density at a height h_n at the beginning of intense evaporation:

$$\rho_n=\frac{8\lambda c_2\left[\frac{r_0}{x_0}\operatorname{cth}\left(\frac{r_0}{x_0}\right)-1\right]}{\Lambda r_0 v^3\left\{-4.23+c_1+\log Q+\frac{1}{2}\log\mu+\log r_0/\lambda+\log\left[\frac{r_0}{x_0}\operatorname{cth}\left(\frac{r_0}{x_0}\right)-1-\frac{3}{2}\log T_n\right]\right\}}, \qquad (27)$$

where T_n is the temperature at the start of intensive evaporation ($T_n \geqslant T_{fusion}$).

With $r_0 \leqslant 2x_0$, according to equation (24), a meteor body at a height of h_n would already be completely molten. Under the influence of aerodynamic pressure and the forces of surface tension, the molten body assumes an

elongated shape. Let us estimate the degree of deformation, assuming that the drop has the shape of an ellipsoid of rotation with the minor axis located in the direction of motion.

The difference in pressures formed by the forces of surface tension in the planes of longitudinal and transverse cross-sections of the drop must be equal to the aerodynamic pressure. By means of the Laplace formula, we will obtain

$$\sigma\left(\frac{a}{b^2}+\frac{1}{a}\right)-\frac{2\sigma}{a}=\Gamma\rho v^2, \tag{28}$$

where σ is the coefficient of surface tension; Γ is the coefficient of resistance; a, b are the major and minor semiaxes of the ellipsoid; $a^2b = r^3$ (r being the radius of a non-deformed drop). Then

$$\left(\frac{a}{r}\right)^3-\frac{r}{a}=\frac{\Gamma r v^2\rho}{\sigma}. \tag{29}$$

In [17] another relationship for the degree of deformation of a drop upon its radius was obtained, having a crack which is practically inexplicable. The minor semiaxis of the ellipsoid, according to Oepik is independent of the radius of the drop.

According to equation (29) the degree of deformation of the drop with change is altered continuously. We can rewrite (29) in the form

$$\frac{r^3}{b^{5/2}}-b^{1/2}=\frac{\Gamma r^{3/2}v^2\rho}{\sigma}.$$

Hence, in contrast to the result obtained by Oepik, the minor semiaxis of the ellipsoid depends on the radius of the drop.

Disregarding the term r/a on the left side of equation (29) (which causes a slight decrease in the degree of deformation), we will have

$$a = \left(\frac{\Gamma v^2 \rho}{\sigma}\right)^{1/5} r^{6/5} \tag{30}$$

The equation for evaporation of the meteor body

$$\frac{dM}{dt} = -\frac{\Lambda}{2(Q-Q_{heat})} S\rho v^3 = \frac{\pi\Lambda}{2(Q-Q_{heat})} v^{19/5} \left(\frac{\Gamma}{\sigma}\right)^{2/5} r^{12/5} \rho^{7/5}, \tag{31}$$

where Q_{heat} is the energy required to heat one gram of meteor substance to the temperature T_{heat}. We can rewrite (31) in the following form for an isothermal atmosphere:

$$\frac{dr}{r^{2/5}} = \frac{3\ Hv_0^{14/5}}{8(Q-Q_{heat})\delta\cos z} \left(\frac{\Gamma}{\sigma}\right)^{2/5} \rho^{2/5} d\rho. \tag{32}$$

/17

Disregarding the braking of the meteor body in the course of evaporation, we will have

$$r^{3/5} = -\frac{3\Lambda H v_0^{14/5}}{56(Q-Q_{heat})\delta\cos z} \left(\frac{\Gamma}{\sigma}\right)^{2/5} \left(\rho^{7/5} - \rho_{heat}^{7/5}\right) + r_0^{3/5} \tag{33}$$

where v_0 is the initial velocity of the meteor body.

From (31) and (33) we find the curve for the light intensity and ionization of the meteor:

$$J = \frac{\tau'\Lambda}{16(Q-Q_{heat})} \left(\frac{\Gamma}{\sigma}\right)^{2/5} v_0^{19/5} \rho^{7/5} \left[r_0^{3/5} - \frac{3\Lambda H v_0^{14/5}}{56(Q-Q_{heat})\delta\cos z} \left(\frac{\Gamma}{2\sigma}\right)^{2/5} \left(\rho^{7/5} - \rho_{heat}^{7/5}\right)\right]^4, \tag{34}$$

$$\alpha = \frac{\pi\Lambda\beta}{2\mu m_{heat}(Q-Q_{heat})} \left(\frac{\Gamma}{\sigma}\right)^{2/5} v_0^{14} \rho^{7/5} \left[r_0^3 - \frac{3\Lambda H v_0^{14/5}}{56(Q-Q_{heat})\delta\cos z} \left(\frac{\Gamma}{\sigma}\right)^{2/5} \left(\rho^{7/5} - \rho_{heat}^{7/5}\right)\right]; \tag{35}$$

Here τ' is the coefficient of luminance and β is the coefficient of ionization.

With sufficiently strong deformation, the drop becomes unstable and breaks up. E. Oepik measured the stability of the molten meteor body on the basis of observations of the breakup of air bubbles in water [17]. This method of evaluation is very approximate, since the fixed conditions for breakup of air bubbles in water have little relationship to the conditions of a movement of a drop of liquid at a velocity of tens of km/sec in a rarefied gas.

As of the present time, a number of theoretical studies have been published dealing with the investigation of the stability of liquid droplets in supersonic gas flows, which show good agreement with the results of experiments [22-26]. The stability of a drop is characterized by the Weber number

$$We = \frac{\Gamma \rho r v^2}{\sigma}. \tag{36}$$

For a non-viscous liquid, the critical value of the Weber number $We_0 \approx 3.2$. For a viscous fluid, $We_0 \approx 6.5$. The drop breaks up at $We \geqslant We_0$.

From (27), (33) and (36) we find the boundary value for the initial radius of the drop r_0^1, retaining stability in the course of evaporation:

$$r_0^1 = \frac{3\sigma We_0}{4\Gamma \rho_h v_0^2}. \tag{37}$$

With $r_0 < r_0^1$, a drop will not break up. In this case, a slight shortening of the meteor track occurs due to the increase in the midship cross-section of the drop due to deformation.

All meteor bodies with $r_0 > r_0^1$ will break up, since:

1) if $1.3r_0^1 \leqslant r_0 \leqslant 2x_0$, breakup will occur at the altitude of the beginning of intensive evaporation;

2) if $r_0^1 \leqslant r_0 < 1.3r_0^1$ and $r_0 \leqslant 2x_0$, breakup will occur when the radius of the drop reaches a critical value corresponding to the condition $We = We_0$;

3) if $r_0^1 \leqslant r_0 \leqslant 1.3r_0^1$ and $r_0 > 2x_0$ breakup will occur as soon as the body melts all the way through, and the radius of the drop reaches a critical value corresponding to the condition; /18

4) if $r_0 \geqslant 1.3r_0^1$ and $r_0 > 2x_0$, breakup will occur as soon as the body melts all the way through (in the case of large meteor bodies, this takes place in the frequently observed final flash of the meteor [27]).

Breakup of a meteor body leads to an increase in the front surface, and consequently to increased rate of evaporation and braking. It is easy to show that when calculating the curve for the luminous intensity of a disintegrating meteor it is possible to disregard the braking of the droplet into which the meteor body breaks up.

Since very fine particles may expend a significant portion of their kinetic energy by turning it into heat radiation, the estimation of the value of braking of the droplets must take into account the energy losses to thermal radiation. The equations for evaporation and braking which take into account the loss of energy to heat radiation are in the following form [28]:

$$\frac{dM}{dt} = -\frac{\Lambda A M^{2/3} v^3}{2(Q-Q_{heat})\delta^{2/3}}\left(\rho - \rho_h''\right), \tag{38}$$

$$\frac{dv}{dt} = -\frac{\Gamma A v^2 \rho}{M^{1/3}\delta^{2/3}}; \tag{39}$$

$$\rho_h'' = \frac{8\eta T_h^4}{\Lambda v^3}, \tag{40}$$

where η is the Stefan-Boltzmann constant, and A is the form coefficient. Dividing (38) by (39) we will have

$$\frac{dM}{dt} = \xi v\left(1 - \frac{\rho_h''}{\rho}\right) dv, \quad \xi = \frac{\Lambda}{2\Gamma(Q-Q_{heat})}. \tag{41}$$

It is easy to see from (27), (37) and (40) that the atmospheric density ρ_h at the altitude where intensive evaporation of the meteor bodies with radii

$r_0 \geqslant r_0^1$ begins, always satisfies the condition $\rho_h/\rho_h'' > 2$. The breakup of molten meteor bodies takes place at an altitude close to that where intensive evaporation begins or below, so that in order to obtain a maximum estimate of the loss of velocity by the droplets during the evaporation process, we can use $\rho_h/\rho_h'' = 1/2$ in (41). Then, integrating (41), we will have

$$\frac{M_0}{M} = \exp\left[\frac{\cdot}{4}(v_0^2 - v^2)\right], \tag{42}$$

where M_0 is the initial mass of the droplet.

Determining the values Q and Q_{heat} from [17] and letting $\Lambda/\Gamma = 1$, we find that with an average meteor velocity $v_0 = 40$ km/sec, regardless of the original mass of the droplet, its reduction of mass (for example by 5 times) will correspond to a reduction of velocity of less than 2%. Since formula (42) gives a maximum estimate for the loss of velocity, the actual braking will be still less. Consideration of such a small decrease in velocity of a droplet in the course of evaporation has no practical influence on the shape of the curve for the luminous intensity of a meteor.

If we do not consider the braking, the curves for the luminous intensity of meteors formed by individual droplets may be calculated by means of formula (34). The integral curve of the luminous intensity is obtained as a result of their summing.

For the sake of an example, let us examine the evaporation of a dense /19
stony meteor body with $M_0 = 0.2$ g, $v_0 = 20$ km/sec, $\delta = 3.4$ g/cm^3 and $\cos z = 2/3$. The radius of the body satisfies the condition 1, so that the breakup must take place at the altitude of the beginning of intensive evaporation. Different forms of breakup are possible: the drop may breakup all of a sudden into very fine droplets, each of which remains stable in the evaporation process, or may undergo progressive breakup. For example, if the drop breaks up into two equal parts, each of them after traveling 2 km will become unstable and will in turn breakup into finer particles. This process, in conjunction with evaporation, will last until the body no longer breaks up into

sufficiently fine stable particles. It is also possible to have a combination of both types of breakup. Figure 1 shows the luminous intensity curve a calculated on the assumption that the drop breaks up into a minimum number of droplets, each of which remains stable in the evaporation process. For comparison, we have also plotted b, the curve for luminous intensity calculated on the basis of (34) without taking breakup into consideration, and c, the curve of luminous intensity given by the simplest physical theory of meteors [29]. The dotted portion of curves a and b correspond to the range of altitudes at which a transition occurs from the regime of heating of the meteor body to the one involving intensive evaporation. The value of the coefficient of luminosity is taken from [17]. Figure 2 shows the similar curves for luminous intensity for a meteor which was produced by an iron meteor body with $M_0 = 0.03$ g, $v_0 = 40$ km/sec and cos z = 2/3.

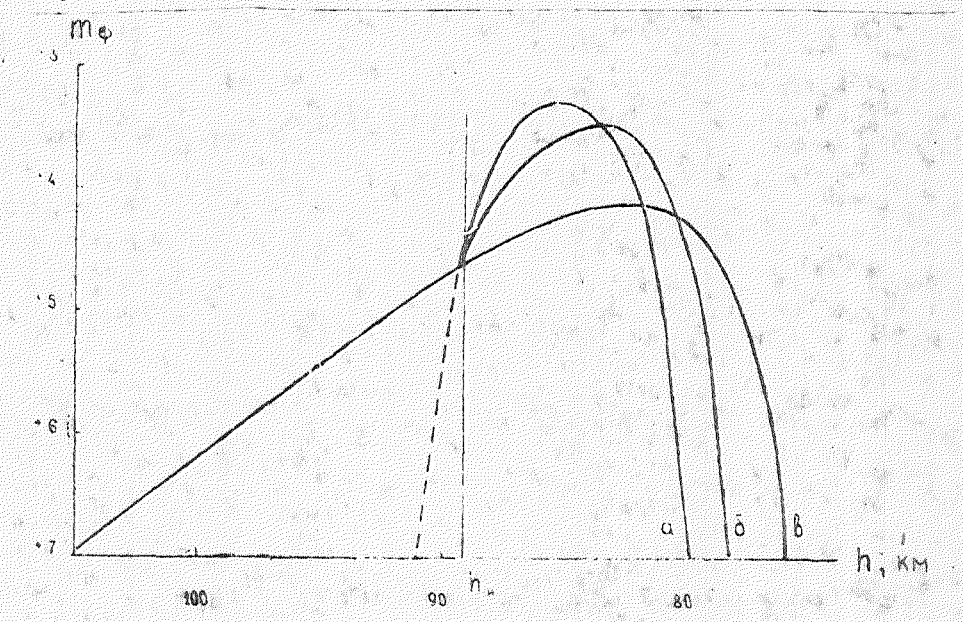

Figure 1.

Consideration of breakup leads to an increase of the maximum brightness of the meteor, a shortening of the track, an increase in braking and an increase in the altitude of maximum evaporation. For example, consideration of breakup leads to an increase in the maximum luminous intensity of a stony meteorite, whose luminous intensity curve is shown on Figure 1 by 0.8 m in comparison to the value given by the simplest physical theory of meteors.

The altitude of maximum evaporation increases by 3 km. On the threshold of sensitivity of the receiver for radiations of +4 stellar magnitudes, made by B. Yu. Levin [1], the breakup of parameter F, which characterizes the relative shortening of the meteor track, is found to be equal to 0.5. Due to breakup and subsequent deformation of the droplets, the coefficient of the form of the meteor reaches a value of 3.6 which leads to an increase in the braking of the meteor by about 3 times in comparison to the case of a spherical meteor body which does not breakup. For an iron meteor, the luminous intensity curve is shown in Figure 2 and the breakup parameter F = 0.3. /20

We have studied the cases of breakup of molten meteor bodies into the minimum number of comparatively large droplets. In the case of breakup into very fine droplets, the effects considered above involving breakup may be much more pronounced.

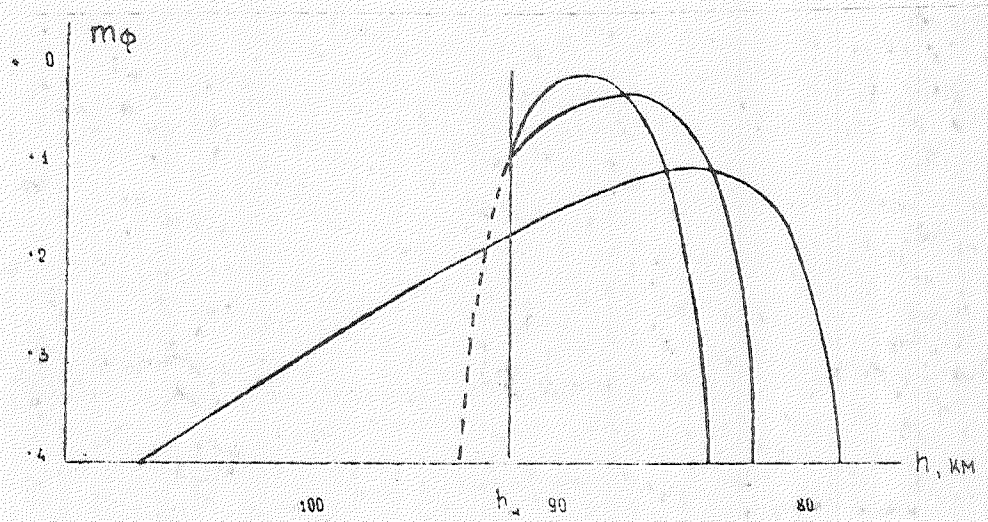

Figure 2.

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